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Price Controls and Consumer Surplus

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*Price controls lead to misallocation of goods and encourage rent-seeking. The misallocation effect alone is enough to ensure that consumer surplus is always reduced by a price control in an otherwise-competitive market with convex demand if supply is more elastic than demand; or when demand is log-convex (e.g., constant-elasticity) even if supply is inelastic. The same results apply both when rationed goods are allocated by costless lottery among interested consumers, and when costly rent-seeking and/or partial de-control mitigates the allocative inefficiency. The results are best understood using the fact that in **any** market, consumer surplus equals the area between the demand curve and the industry marginal revenue curve.*

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1 Introduction

Price controls and rent controls, though clearly inefficient in competitive markets, do increase consumer surplus in the short run. Over the longer run, consumer surplus falls for three reasons. The first, emphasized in every textbook, is that long run supply is reduced—indeed some textbooks’ analyses focus exclusively on this as a reason why consumers might lose from price controls.¹ The second reason, discussed since at least Friedman and Stigler (1946) but rarely emphasized until Glaeser and Luttmer (1997, 2003), is that the available supply will not necessarily be allocated to those with the highest value.² The third reason is that non-price allocation mechanisms can lead to costly rent-seeking behavior, such as queuing, lobbying and search costs. So under what conditions do price controls reduce consumer surplus?

We show that if output is allocated randomly among those prepared to pay more than the controlled price and if supply is more elastic than demand, then a price control always hurts consumers if demand is convex (e.g., linear, log-linear, etc.). Even with completely inelastic supply, total consumer surplus falls whenever demand is log convex (constant elasticity is one example).

Furthermore, these results are unaffected if rent-seeking affects the allocation. Though rent-seeking leads to more-efficient-than-random allocation, the costs it dissipates mean a price control is guaranteed to hurt consumers under the identical conditions.³

Splitting the market between controlled and uncontrolled units also makes no difference to these results. Even though all the highest-value consumers can consume, the results are the same (although the magnitudes of con-

¹For example, the analyses in Taylor and Weerapana (2007, p193) and Boyes and Melvin (2010, p518-9), which are widely-used textbooks in US universities and colleges, simply assume efficient allocation without discussion.

²Lott (1990), Luttmer (2007), and Palda (2000) discuss allocative costs in the context of minimum-wage legislation; and MacAvoy and Pindyck (1975), Braeutigam and Hubbard (1986), and Davis and Kilian’s (2011) careful recent study analyse the costs of restricting new potential consumers’ access to the natural gas market. A clear exposition of the standard theoretical analysis of these "allocative costs" is in Viscusi, Harrington, and Vernon (2005).

³We show below that there are (other) conditions under which rent-seeking does change the sign of rationing’s effect on consumers’ welfare, and its *quantitative* importance depends on the distribution of consumers’ costs of rent-seeking; our results also depend on these costs being uncorrelated with valuations, as we also discuss later.

sumers' losses are, of course, affected).⁴

Finally, while there is always a windfall gain to incumbent consumers, these gains are often small. For example, below-market rents are typically phased in over time by rent freezes rather than cuts, and turnover in rentals is on average very high. So even when controls raise surplus in the short run because of the incumbent effect, and even though a gradual implementation of rent control will also mean a slower decline in the value of the marginal rental seeker, the mis-allocation effect alone can quickly cause a net loss in consumer surplus.

Existing analyses fail to note that *consumer surplus equals the area between the demand curve and the industry marginal-revenue curve* up to the market quantity in an uncontrolled market—even in a competitive market. More generally, when goods are rationed, total consumer surplus equals the sum of the values to consumers of the units they receive less the sum of their marginal revenues.⁵ *These facts are the key* to the development and interpretation of our results.

One caveat is that our analysis ignores distributional issues—even when price controls reduce aggregate consumer surplus, they redistribute it among consumers.

We begin in Section 2 by assuming that output is allocated randomly among those willing to pay more than the fixed price, before extending the model to address non-random allocation. Section 3 considers the effects of rent-seeking, of partially-controlled markets, and of secondary markets. Section 4 illustrates our analysis by solving our model for a class of examples that includes the standard constant-elasticity, log-linear, and linear demand curves, and Section 5 concludes.

⁴Examples of partially controlled markets include Manhattan real estate (where some units are either rent-controlled or rent-stabilized while others are available to the highest bidder), and the British healthcare system (where a small private market coexists with a National Health Service which approximates random rationing since healthcare professionals are roughly uniformly distributed across the population).

⁵For example, if the inverse demand curve were $p = 100 - q$ and so $MR = 100 - 2q$ then a consumer with a value of 70 (and so MR of 40) and a 50 percent chance of receiving a unit would account for $.5(70 - 40) = 15$ in consumer surplus, and aggregating across all consumers in this way correctly calculates total consumer surplus, even though the measure cannot be used for determining the amount of consumer surplus that goes to the individual consumer (which, of course, also depends on the market price).

2 The Basic Model: Rationing by Lottery

Consider a competitive industry with a demand curve $D(p)$ formed by a density of consumers $-D'(v) \geq 0$ with unit demand at value v ,⁶ and a supply curve $S(p)$.

We assume $S'(p) \geq 0$ (that is, no "backward-bending" supply). We also assume that demand is finite at all $p > 0$, and that its elasticity at all prices above some finite price is bounded strictly below 1. This condition ensures that total consumer surplus in an uncontrolled market that clears at price p (that is, $\int_p^\infty D(v)dv$) is finite. (So, for example, constant-elasticity demand with inelastic demand is ruled out.⁷)

We assume in Sections 2.1-2.2 that if a regulator sets a price p below the market clearing level then demand is randomly allocated among consumers with value $\geq p$. This is the standard assumption made in, for example, Viscusi, Harrington, and Vernon (2005), but we will relax it in subsequent Sections of our paper.

So consumer surplus at the controlled price, $CS(p)$, equals consumer surplus if the market cleared at p , times the ratio of supply to demand, $S(p)/D(p)$:

$$CS(p) = \frac{S(p)}{D(p)} \left[\int_p^\infty D(v)dv \right]. \quad (1)$$

2.1 Measuring Consumer Surplus using Marginal Revenues

For any quantity in any market, price times quantity equals total revenues equals the area under the monopolist marginal revenue (MR) curve. Therefore, if all consumers with values above p are served, *consumer surplus equals the area between the demand curve and the marginal revenue curve, in **any***

⁶We discuss the extension of our results to demands in which consumers individually have downward-sloping demand curves in Section 3.

⁷Our assumptions also ensure that a monopolist's problem is well-defined—although our model is a competitive one, we will see below that the demand conditions we develop can be related to the conditions that determine a monopolist's rate of pass-through.

market, including our competitive one. So we can rewrite (1) as

$$CS(p) = \frac{S(p)}{D(p)} \int_p^\infty -D'(v)[v - MR(v)]dv \quad (2)$$

since $-D'(v)$ is the density of consumers with a value of v .⁸

Differentiating with respect to price yields the change in consumer surplus due to a small price cut:

$$-CS'(p) = -D'(p) \frac{S(p)}{D(p)} [MCS(p)] + [D'(p) \frac{S(p)}{D(p)} - S'(p)] [ACS(p)] \quad (3)$$

in which $MCS(p) \equiv [p - MR(p)]$ is *Marginal Consumer Surplus*, that is, the increment in total consumer surplus in an uncontrolled market caused by a price reduction that leads to a one unit increase in quantity, while $ACS(p) \equiv \left[\frac{\int_p^\infty -D'(v)[v - MR(v)]dv}{D(p)} \right]$ is *Average Consumer Surplus*.

Since $S'(p) \geq 0$, consumer surplus must decline if $ACS(p) > MCS(p)$.

The intuition is trivial: with random allocation of a fixed number of units, consumer welfare is proportional to *Average CS*, which of course declines if $Average CS > Marginal CS$. If supply falls with price, that only reduces consumer welfare further.

Now any log-convex demand curve has the same $MCS(p)$ as the log-linear demand that is tangent below it at p (because $MCS(p) = MR(p) - p = -D(p)/D'(p)$). And the log-convex demand clearly has higher $ACS(p)$ (since it has weakly higher $D(p)$ everywhere). But $ACS(p) = MCS(p)$ for any log-linear demand, because $MCS(p)$ is constant for log-linear demand (since $D(p) = \exp((\alpha - p)/\beta)$, i.e., $p = \alpha - \beta \log(D(p))$, gives $MCS(p) = -D(p)/D'(p) = \beta$, a constant). Therefore $ACS(p) > MCS(p)$ for all log-convex demand, so⁹

Proposition 1: *When a rationed good is allocated randomly, consumer surplus is always reduced by a tighter price control if demand is log-convex.*

⁸We can derive (2) directly from (1) using $MR(v) \equiv v + D(v)/D'(v)$ (the derivative of total industry revenue $vD(v)$ with respect to quantity $D(v)$ in an uncontrolled market with price v).

⁹Our subsequent analysis is based on equation (2), but this Proposition can be obtained directly from (1): differentiating with respect to price yields $-CS'(p) = S(p) - S'(p) \frac{\int_p^\infty D(v)dv}{D(p)} + D'(p) \frac{S(p)}{D^2(p)} \left[\int_p^\infty D(v)dv \right]$. Now if demand $D(p)$ is log-convex then, since also $D(\infty) = 0$, $\int_p^\infty D(v)dv$ is also log-convex (see, e.g., Bagnoli and Bergstrom, 2005, Thm 4), that is, $(-D'(p))(\int_p^\infty D(v)dv) \geq (D(p))^2$. So, since $S'(p) \geq 0$, the result follows.

Figures 1A and 1B illustrate the analysis. Fig. 1A measures consumer surplus conventionally, as in equation (1). Consumer surplus at the market price p^{Market} is the heavily-shaded area, and the effect on consumer welfare of reducing price to a controlled level $p^{Control}$ would be the sum of areas A (the benefits to existing buyers) and B (the benefits to new buyers), *if* supply could expand from $D(p^{Market})$ to meet the new level of demand $D(p^{Control})$. So with random allocation of demand, the average consumer surplus per consumer served equals the average height of the whole area formed by both the shaded areas together.

So Fig. 1A suffices to show that if demand is sufficiently "fat-tailed", then average consumer surplus is decreasing in the price control, so rationing hurts consumers. But Fig. 1B tells us *how* fat-tailed.

In Fig. 1B we have drawn the MR curve onto Fig. 1A. Because, of course, the area under the MR curve up to $D(p^{Market})$ equals total revenue at that quantity, the labelled areas satisfy $X + A1 + Y = A1 + Y + A2 + Z$. So $X = A2 + Z$, and the heavily-shaded areas in Figs 1A and 1B are therefore equal and both represent consumer surplus at the market price, p^{Market} . Likewise, the area under the MR curve up to $D(p^{Control})$ equals total revenue at that quantity, so the sum of the heavily- and lightly-shaded areas in Figs 1A and 1B are also equal and would both represent consumer surplus at the controlled price, $p^{Control}$, *if* all the demand at that price could be satisfied. The lightly-shaded areas in Figs 1A and 1B are therefore equal as well, and represent the incremental welfare from reducing the price *if* supply could expand to meet the incremental demand.¹⁰ So if the average height of the heavily-shaded area exceeds that of the lightly-shaded area in Fig. 1B, i.e., $ACS(p) > MCS(p)$, then *Average CS* falls, and therefore total consumer welfare also falls, even with no fall in supply.

Finally, since $MCS(p) = p - MR(p)$, consumers are hurt by price controls if marginal revenue is steeper than demand, that is, for any log-convex demand, such as, for example, constant-elasticity demand.¹¹

¹⁰Of course only area B of the incremental consumer surplus goes to the new purchasers; the area $A + B = (A1 + A2) + B = C + B$ is the amount of surplus gained by *all* consumers when price falls by enough to attract $D(p^{Control}) - D(p^{Market})$ additional purchasers.

¹¹Though the market we are modelling is competitive, our condition for a price reduction to hurt consumers ($ACS(p) > MCS(p)$, or equivalently demand is log-convex) also has simple monopoly-theory interpretations: it is the condition for the constant-marginal-cost monopolist, that would set this price, to generate greater consumer surplus than profits (because its per-customer profit $= p - AC = p - MC = p - MR(p) = MCS(p)$). It is

2.2 Elastic Supply

Dividing the right-hand side of (3) by $S(p)/p$ yields

$$\text{sign}[-CS'(p)] = \text{sign}[MCS(p) |\text{Elasticity of Demand}| - ACS(p)(\text{Elasticity of Supply} + |\text{Elasticity of Demand}|)]$$

So if the elasticity of supply is greater than or equal to (the absolute value of) the elasticity of demand, consumers always lose if $MCS(p) < 2ACS(p)$. But every linear demand curve satisfies $MCS(p) = 2ACS(p)$ (since MR is twice as steep as demand), and the linear demand curve that is tangent to any convex demand curve at p has the same $MCS(p)$ and lower $ACS(p)$. So we have

Proposition 2: *When a rationed good is allocated randomly, consumer surplus is always reduced by a tighter price control if supply is locally more elastic than demand and demand is convex.*

So a pass-through rate of 50% or more in a competitive industry with convex demand would imply consumers lose from a price control. (In a competitive market, pass-through = [*elasticity of supply* / (*elasticity of supply* + *|elasticity of demand|*)].) Campa and Goldberg's (2005) study based on exchange-rate changes estimates short-run and long-run pass-through for 23 countries at .46 and .64, respectively, and other studies using exchange-rate changes obtain similar results. Results such as these suggest that whether a small regulated price cut would benefit consumers is likely to vary from market to market.¹²

also the condition for such a monopolist to pass through $> 100\%$ of any (marginal) tax or cost increase (because its pass-through rate = $\frac{dp}{dMC} = \frac{dp}{dMR} = \frac{\text{slope of demand}}{\text{slope of MR}}$ (Bulow and Pfleiderer, 1982) and it is easy to see from Fig. 1B that if the slope of demand (always) exceeds that of MR , then $ACS(p) > MCS(p)$). Weyl and Fabinger (2009) show the pass-through result extends to a very broad class of Cournot oligopoly contexts; see also Weyl and Fabinger (2011), and the 2008 version of our current paper. Our result is also analogous to Spence's (1975) result that a monopolist over- or under-provides quality depending on whether the marginal value of quality is higher for the marginal or the average consumer.

¹²Oligopolistic industries may have lower pass-through than competitive ones, so these results may understate average pass-through in competitive markets and so overstate consumers' expected benefit from a price control.

Economists tend to assume demand is convex, although relatively little is known about actual functional forms—see, e.g., Blundell, Browning, and Crawford (2008) and the references they cite.

2.3 Incumbent Consumers vs. Newcomers

Thus far we have focused on the long-run distributional consequences of price controls. But for durables such as rental apartments the inefficiencies created by a price control will phase in gradually, so even if new tenants receive less consumer surplus on average after controls are implemented, there is a group of incumbents who receive a windfall transfer from the lower prices. Following Glaeser and Luttmer (1997), we can model this by assuming that if supply with a price control is S and demand is D then the S buyers with the highest values buy with probability $\lambda + (1 - \lambda)S/D$ while the remaining $D - S$ buy with probability $(1 - \lambda)S/D$ (so if $\lambda = 1$, the rationing is perfectly efficient).

Clearly if enough of the supply is allocated efficiently, and without any reduction of supply, consumer surplus must rise. However, the conditions for consumer surplus to fall still do not seem onerous. An argument paralleling that of the previous subsection (see Appendix A) shows that *for any convex demand, consumers always lose from a small price cut below the uncontrolled market price if supply is at least $\frac{1+\lambda}{1-\lambda}$ as elastic as demand; or for any log-convex demand, if supply is at least $\frac{\lambda}{1-\lambda}$ as elastic as demand.* And we show in Appendix B that with demand of constant-elasticity η , and (any functional form of) supply with elasticity φ , consumers always lose from a small price cut below the market price if $\lambda > \frac{\varphi+1}{\varphi-\eta}$.¹³

Furthermore, this model assumes prices are immediately reduced when the control is announced. More commonly, price controls are phased in only gradually by restraining price increases to below-market rates. Also, turnover is on average high in markets such as that for rental accommodation. Both these things reduce the relative importance of the incumbents' windfall.¹⁴ So even when controls raise surplus in the short run because of the incumbent effect, the misallocation effect alone can quickly cause a net loss in consumer surplus. We illustrate this in Appendix C.

¹³Appendix B also generalises the allocation process further by assuming an additional fraction of supply is allocated as inefficiently as possible above the controlled price—this case is obviously extreme, but Glaeser and Luttmer (1997) point out, for example, that long-time residents may have greater access to, but less desire for, rent-controlled apartments, than transients.

¹⁴A gradual implementation of price control does also mean a slower decline in the value of the marginal consumer, but some consumers with lower values than the current price jump in straight away to capture the expected gains from being an incumbent in the future.

3 A Model of Rationing with Rent-Seeking

Our basic model in which all consumers who wish have an equal chance of being able to buy at the controlled price, with no additional search or rent-seeking costs, is a special case of a more general model in which consumers expend “effort” competing for the rationed good:

Let each consumer have a marginal cost of effort drawn from an arbitrary distribution, independent of the consumer’s value. (We will generalise this later.) A consumer’s probability of purchase is proportional to the effort it makes. Competition determines the probability of purchase per unit of effort expended: if E is the sum of all consumers’ efforts, then $E/S(p)$ of effort earns one unit. So a consumer who has marginal cost of effort c chooses effort $E/S(p)$ if his value $v \geq p + cE/S(p)$, and expends no effort (and does not buy the good) otherwise. This condition determines the total effort, E , expended in equilibrium, and hence the equilibrium allocation of the goods.^{15,16}

Let $n(v)$ be the expected quantity per consumer bought by consumers with value v , and $cs(v)$ be the expected surplus per consumer of *these* consumers, in equilibrium.

The standard mechanism-design argument [see, e.g., Myerson (1981)], using the envelope theorem, then tells us $dcs/dv = \partial cs/\partial v = n(v)$. (Since each consumer chooses its effort, and so purchase quantity, optimally, each of $n(v)$ consumers with values $v + dv$ obtains dv more surplus than the otherwise-identical consumer with value v who obtains a unit, but the total surplus of the additional dn consumers with value $v + dv$ who would not have purchased units if their value were just v is second order.) Also $cs(p) = 0$, so $cs(v) = \int_p^v n(x)dx$.¹⁷

¹⁵To see equilibrium is generally unique, observe a proportional increase in *anticipated* E yields the same proportional increase in effort for those consumers who still purchase, but reduces the number of purchasers, so yields a smaller than proportional increase in actual E .

¹⁶Since our risk-neutral consumers want at most one unit each, nothing would change if we assume a single unit is allocated to each of the $S(p)$ consumers who make the greatest effort. (Technically, there is then no equilibrium if consumers make simultaneous effort choices, but the outcome in the text is the equilibrium if consumers make sequential choices; it is also the limit of equilibria of discrete versions of the simultaneous game.)

¹⁷The function $n(v)$ itself depends on p , but we suppress this dependence for notational simplicity. Also note that supply $S(p) = \int_p^\infty -D'(v)n(v)dv$.

Integrating across all consumers, total consumer surplus at price, p , is

$$CS(p) = \int_p^\infty -D'(v)cs(v)dv = \int_p^\infty -D'(v) \int_p^v n(x)dx dv$$

so, integrating by parts and observing $\left[D(v) \int_p^v n(x)dx \right]_p^\infty = 0$,¹⁸ we have

$$CS(p) = \int_p^\infty D(v)n(v)dv = \int_p^\infty -D'(v) \left[v - \left(v + \frac{D(v)}{D'(v)} \right) \right] n(v)dv.$$

Writing $MR(v) \equiv v + \frac{D(v)}{D'(v)}$ for the marginal revenue at value v of a monopolist on the demand curve $D(v)$, as before, therefore gives us

$$CS(p) = \int_p^\infty -D'(v) [v - MR(v)] n(v)dv \quad (4)$$

which is just the generalisation of our result in equation (2) above. From consumers' point of view, rent-seeking costs simply increase (by differing amounts) the "effective price"s they face. The fact that the rent-seeking part of these effective prices is a social waste is irrelevant to them. So, just as before, total consumer surplus can be computed by summing the values less the marginal revenues of all those who actually purchase.

Figure 1B illustrates this. Total consumer surplus is just the integral of the shaded area but with each strip of height $v - MR(v)$ (and of width equal to the density of consumers with value v —i.e., $-D'(v)$) is weighted by the total number of units, $n(v)$, that bidders with value v get. Thus Figure 1B/equation (4) allows the computation of consumer surplus knowing only the probabilities with which different types of consumers receive units. (Of course, a consumer's own per-unit surplus is *not* equal to the height, $v - MR(v)$, of "its" strip—the calculation applies only in aggregate.)

By contrast, the corresponding integral of the shaded area in figure 1A measures consumer welfare only in our Basic Model (section 1), and *not* in our General Model, since only the random-allocation case can arise without rent-seeking activities; the height, $v - p$, of type v 's strip in figure 1A shows

¹⁸ $\left[D(v) \int_p^v n(x)dx \right]_p^y = D(y)cs(y) < D(y)y$ and $\lim_{y \rightarrow \infty} D(y)y = 0$ by our earlier assumption that the elasticity of demand is bounded strictly below 1 at all sufficiently high prices.

its per-unit gross surplus *ignoring* any resources it spends to increase its probability of winning above that of a consumer of value p .¹⁹

The implication is that if the demand curve is always steeper than the MR curve, so $(d/dv)[v - MR(v)] > 0$ (and also $MCS(p) < ACS(p)$ —see Figure 1B—so demand is log-convex), then any transfer of probability from a higher- v consumers to a lower- v consumer reduces consumer welfare.

Furthermore, a tighter price control always results in some high- v consumers being displaced by low- v consumers, and not vice versa (because the equilibrium amount of effort required to obtain a unit is increased, so high- v consumers who did not buy previously are more disadvantaged relative to any low- v consumers who did and who must therefore have lower rent-seeking costs). So, since any supply response only reduces consumer welfare further,²⁰ we can generalise Proposition 1:

Proposition 3: *When a rationed good is allocated by rent-seeking, consumer surplus is always reduced by a tighter price control if demand is log-convex.*

Conversely, if $(d/dv)[v - MR(v)] < 0$ everywhere, so $MCS(p) > ACS(p)$, then a price control that does not cause a supply cut must increase consumer surplus. Because rent-seeking costs that are uncorrelated with values cannot lead to more substitution of high- v by low- v consumers than in a random allocation, no distribution of rent-seeking costs can yield greater consumer surplus than a random allocation at the same price. This can arise when at least the fraction $S(p)/D(p)$ of consumers have zero costs of rent-seeking; since no rent-seeking costs are then actually incurred, this corresponds exactly to our basic model of random rationing in Sections 2.1-2.2.

¹⁹However, this integral, $\int_p^\infty -D'(v)[v - p]n(v)dv$, does show the consumer surplus that could be achieved by an informed principal who could allocate higher probabilities to favoured types unconstrained by any need to impose greater costs (either through the price charged, or through deadweight rent-seeking activities) on the favoured consumers. (In the same way, the revenue of an ordinary monopolist—including one that sets different prices at which consumers can buy goods with different probabilities—is the sum of the MR s of the consumers it sells to, but the revenue of a monopolist that can somehow price discriminate costlessly is the sum of the maximum willingnesses-to-pay of the consumers it sells to.)

²⁰The effect of a price cut can be divided into the effect which would occur were supply inelastic, and a supply effect. The latter effect always reduces consumer surplus (by the sum of $v - MR > 0$ across all the consumers it displaces, since no consumers buy as a result of the supply effect who would not buy in its absence).

Note that because rent-seeking increases the efficiency of the allocation (less substitution of high- v by low- v consumers than in a random allocation), it reduces consumers' losses from rationing when demand is log-convex, but reduces their gains from rationing when demand is log-concave and supply is inelastic.

In the extreme case, if all consumers have identical costs of rent-seeking activities, the available supply is efficiently allocated to the highest-value consumers, exactly as in an uncontrolled market, but (with inelastic supply) the entire price reduction is eaten up by the rent-seeking activity—so consumer welfare is unaffected. (And if there is any supply response at all, the "effective price" to consumers rises—that is, more than the entire price reduction is eaten up by the rent-seeking activity, and consumer welfare is reduced.)

So with log-convex demand a price-control is always bad news for consumers, though less so with rent-seeking, while with log-concave demand rent-seeking reduces any benefits to consumers.²¹

3.1 Elastic Supply with Rent-Seeking

With inelastic supply, and either log-convex or log-concave demand, rent-seeking always dampens but never reverses the effect on consumers of imposing a price-control. But the supply response to a price control (which always hurts consumers²²) is, of course, independent of whether or not there is rent-seeking. So, when demand is log-concave and supply responds to price, the overall effect on consumer surplus may turn from positive with random rationing to negative with rent seeking.

We can therefore generalise our earlier proposition about any convex demand (including mixtures of *log-concave* and *log-convex*) when supply is elastic. To do this, note that since $MCS(p) = -D(p)/D'(p)$, we have $(\frac{-D(p)}{MCS(p)})' =$

²¹If demand has both a log-concave section *and* a log-convex section at higher prices, then rationing with rent-seeking *may* help consumers even when random rationing would hurt them, if new consumers displace low- v low- MCS consumers, but would displace a mixture of these and higher- v higher- MCS consumers with random rationing. For example, with sufficiently inelastic constant-elasticity demand at high prices, linear demand at lower prices, and sufficiently inelastic supply, we can find market- and controlled-prices on the linear part of demand that lower consumer surplus with random allocation, but raise it if half of consumers have no rent-seeking costs while the other half have identical (positive) costs.

²²See note 20.

$D''(p) \geq 0$ if demand is convex. So convexity implies $\frac{MCS(p)}{MCS(\tilde{p})} \geq \frac{D(p)}{D(\tilde{p})}$ if $p > \tilde{p}$. Combining this fact with the method we used to show Proposition 2 of considering the linear demand that is tangent to any given convex demand at the market price, allows us to extend that proposition to show (see Appx D)

Proposition 4: *When a rationed good is allocated by rent-seeking, consumer surplus is always less than in an uncontrolled market if supply is more elastic than demand between the uncontrolled market price and the price control and demand is convex.*

Observe, however, that this proposition discusses only the *total* effect of a price reduction from the market-clearing price (so is only a partial generalisation of Proposition 2). The reason is that the amount by which rent-seeking increases the allocation efficiency can vary substantially as the controlled price changes.²³ So the rate of change of the non-supply consumer benefits from rationing, and therefore the extent to which they can outweigh supply effects, can also vary substantially as the price control changes.²⁴ So it is *not* true that any *marginal* tightening of an existing price control necessarily makes consumers worse off under the conditions of Proposition 4.²⁵

3.2 Partially-Controlled Markets

Our results are unaffected if only a fraction of inelastically supplied goods are sold at a controlled price, while the remainder are sold on the free market—because allowing consumers to pay a price premium for an uncontrolled unit is equivalent, from their point of view, to selling all the units at the controlled price but capping their rent-seeking costs. (The cap would be such that in equilibrium a consumer’s total rent-seeking costs of obtaining a unit would

²³For example, with the distribution of rent-seeking costs described at the end of note 21, rent-seeking makes the allocation substantially more efficient than random when supply is 3/4 of demand, but has no effect at all on the allocation after price has fallen to where supply is 1/2 of demand, i.e., the effect of rent-seeking (and the amount spent on it) may *fall* as price falls.

²⁴This does not affect Proposition 3, since a tighter price control then always reduces consumer welfare, independent of supply effects.

²⁵For example, with linear demand and supply as elastic as demand, a small tightening of the price control strictly benefits consumers at the price at which supply is 2/3 of demand (but would be neutral with random rationing), if the distribution of rent-seeking costs is as described at the end of note 21.

be not more than the (equilibrium) difference between the controlled price and the free-market price.)

A special case is a market in which some units are sold at a controlled price *without any* rent-seeking costs (i.e., using a costless lottery), while the rest are sold on the free market. From a consumer surplus point of view, this corresponds exactly to a fully controlled market in which some consumers (those who would succeed in the lottery) have no rent-seeking costs, while the others, who correspond to the buyers of free market units, all have equal, positive costs. Those buyers then clear the market at a total cost of effort plus controlled price which equals what would be the clearing price for the de-controlled units.

Likewise, our results generalize further to cases where, as in cities such as New York, price controls vary by unit, with some units at the minimum controlled price, some at higher but still constrained prices, and some at unconstrained prices. One can think of the units being sold off for varying packages of money and search effort, with each consumer acquiring whatever is cheapest for him given his cost of effort (see Appendix E).

In all these cases, consumer surplus can still be calculated as $CS(p) = \int_p^\infty -D'(v)[v - MR(v)]n(v)dv$, that is, the integral of (value *minus* MR) weighted by the number of consumers of each value who will receive a unit. The integral of MR alone (with the same weights) is the sum of the moneys consumers spend on controlled plus uncontrolled units *plus* the value of the rent-seekers' efforts valued at the costs the rent-seekers themselves attribute to these activities.²⁶

3.3 Rent-Seeking Costs Correlated With Values

If consumers with higher values for the good have higher costs of effort, this can obviously reduce welfare further. If the distributions of effort costs for consumers with higher values for the rationed good first-order stochastically dominate the distributions for lower-value consumers, then consumer surplus must be lower than if all consumers' costs were drawn from any common distribution (independent of value) that yields the same allocation of the

²⁶Rent-seeking and partial decontrol may have very different long-run supply effects. For example, reducing rents on existing housing, but credibly committing to never interfering with new housing, must help consumers if the supply of new housing is perfectly elastic, since they can always rent a new unit at the market price.

good.²⁷ Of course, if effort costs are proportional to $(v - p)$, then consumer welfare at the rationed price, p , is zero.

Conversely, if higher-value consumers have lower costs of effort this increases welfare. However, only if all consumers whose values exceed the uncontrolled market-clearing price can acquire units with zero effort costs will we obtain the traditional textbook outcome with neither misallocation nor rent-seeking costs.

3.4 Other Extensions

A range of other generalisations and extensions of our results are straightforward.

It is trivial to generalise from our model in which each consumer has a constant marginal cost of effort (drawn from some distribution) to one in which consumers have general cost functions for effort (drawn from a set of cost-of-effort functions). Exactly as before we let $n(v)$ be the expected quantity per consumer bought by consumers with value v ,²⁸ and $cs(v)$ be the expected surplus per consumer of these consumers, in equilibrium, and the envelope theorem tells us $dcs/dv = \partial cs/\partial v = n(v)$, for the same reasons as before, etc., so the results are identical. This case, too, can be extended to consumers with different values having different distributions of cost-of-effort functions.

We have modelled demand as comprised of consumers who have different values for a single unit each,²⁹ but the same results apply when demand consists of consumers who have downward-sloping demands that are identical,

²⁷Let the distribution of type v 's effort cost be $F_v(\cdot)$. Assume the type, $\hat{v}(c)$, which purchases if and only if its effort cost $\leq c$, is strictly increasing (otherwise there is in general no common distribution that yields the same allocation of the good). If $F_v(\cdot)$ first-order stochastically dominates $F_{v'}(\cdot) \forall v > v'$, substituting $F_{\hat{v}(c)}(\cdot)$ for $F_v(\cdot) \forall v$ changes neither the allocation, nor the aggregate effort expended by the set of consumers of any type, but reduces the aggregate *costs* of the effort expended by every such set of consumers.

²⁸Parallel to our basic model of rent-seeking, write E for the anticipated sum of *all* consumers' efforts. A consumer who has value v , and cost-of-effort function $c(e)$, chooses an effort, e , that maximises $(eS(p)/E)(v - p) - c(e)$ s.t. $eS(p)/E \leq 1$. Since a larger E yields an e/E that is lower for all consumers, and strictly lower for some, there is a unique E for which the integral over all consumers of e/E equals 1, as required for equilibrium. This E yields a unique distribution of effort levels (consumers mix if their optimal choices are not unique) and a unique equilibrium allocation, $n(v)$.

²⁹Examples might include rental housing, healthcare, and minimum wages.

or proportional to each other. (As before, consumers have differing constant "effort" costs, and competition determines the equilibrium quantity of effort required to obtain one unit of the good, so it also determines each consumer's per-unit effort cost and the effective per-unit price the consumer faces.)

To see why the results are unchanged, observe that constraining the market price is equivalent to breaking the market into submarkets, each of which is identical or proportional to the others, but each of which has a different "effective price" corresponding to its consumers' equilibrium per-unit cost of effort. So if total quantity is fixed, making the change from an uncontrolled price (so search costs are zero for all, and effective prices are identical) to a controlled price (so search costs and therefore effective prices differ) reallocates some units from higher-value to lower-value uses. So consumer welfare is reduced if $p - MR$ is decreasing in price, that is, if demand is log-convex. Also as before, we exactly replicate Section 1's lottery model if a fraction $S(p)/D(p)$ of consumers has no rent-seeking costs (or alternatively the complementary fraction has infinite costs). In this case each consumer is either fully served at the controlled price or not served at all, so the inefficiencies and welfare losses result from overconsumption by those lucky enough to be served. Supplies of natural gas are an example (see, for example, Davis and Kilian's (2011) recent study).³⁰

3.5 Secondary Markets

With inelastic supply and frictionless resale, consumers weakly benefit from any price control, because the secondary market price will equal the original market price and some consumers will get units more cheaply. These consumers essentially earn a middleman's profit by "reselling" to themselves, but others will enter the market purely to resell. If the initial allocation is by a costless lottery, the sum of consumer and reseller surplus equals the consumer surplus in the simple Econ 1 diagram which assumes costless efficient

³⁰If consumers have *decreasing* average costs of rent-seeking, then their welfare losses are *even greater* than in our model with constant marginal and average rent-seeking costs. For example, the rent-seeking cost of queueing for tickets might be independent of the number bought. On the other hand, consumers with increasing rent-seeking costs will have lower losses; for example, limiting the number of tickets any consumer can buy creates an infinite marginal cost at the limit—the allocation of food during wartime might be an example of rationing that is more-efficient than in our model.

allocation. But rent-seeking by middlemen can compete away all their profits, and (with inelastic supply) efficient resale will then recreate the market allocation, that is, fully undo the effects—whether positive or negative—of any price control.

If consumers have differing costs, independent of their values, of participating in the resale market—e.g., legal evasion costs, if the market is "black"—resale will still improve upon a random initial allocation, but it will not be fully efficient. The final allocation may be less efficient than would occur with rent-seeking and no resale. So the entry of additional middlemen combined with an inefficient secondary market might reduce both aggregate buyer value and aggregate surplus.

4 Example

We illustrate our results for the standard distributions of demand—including linear, log-linear, constant-elasticity, etc.—in the class of Generalized Pareto distributions (GPDs). For GPDs

$$D(p) = k \left(1 + \frac{\xi(p - \mu)}{\sigma} \right)^{-1/\xi}$$

($\xi = -1$ gives linear demand, $\xi \rightarrow 0$ gives log-linear demand, and $\xi = \sigma/\mu > 0$ gives constant elasticity demand with elasticity $-1/\xi$.³¹) We write $\eta (= \frac{pD'(p)}{D(p)} = \frac{-p}{\sigma + \xi(p - \mu)})$ for the elasticity of demand at p . We do not restrict the functional form of supply, but write φ for its elasticity at p .

Demands in the GPD class have the useful property that $MR(p)$ is affine in p , since $MR(p) = p + D(p)/D'(p) = \xi\mu - \sigma + (1 - \xi)p$. In particular, therefore, $E\{MR(x)\} = MR(E\{x\})$, for any distribution of x .

Effect of a Price Control: From equation (4) total consumer welfare is $CS(p) = \int_{v=p}^{\infty} -D'(v)n(v)[v - MR(v)]dv$. Equivalently, writing \bar{v} and $\overline{MR}(v)$ for the expected value and the expected MR , respectively, of consumers who get units, $CS(p) = S(p)[\bar{v} - \overline{MR}(v)] = S(p)[\bar{v} - MR(\bar{v})]$ (since $MR(v)$ is affine in v for GPDs).

³¹Our model requires $\xi < 1$ so that consumer surplus is finite. As $\xi \rightarrow 0$ the GPD becomes $D(p) = ke^{(\mu - p)/\sigma}$ with $\sigma > 0$.

But, writing \bar{c} for the expected amount per-unit spent on rent-seeking (priced at the cost to the consumers who expend the effort), we can also write $CS(p) = S(p) [\bar{v} - (p + \bar{c})]$. So we have $p + \bar{c} = MR(\bar{v}) = \xi\mu - \sigma + (1 - \xi)\bar{v}$, so also $\bar{v} = \frac{1}{1 - \xi} [\sigma + p + \bar{c} - \xi\mu]$. Substituting this expression for \bar{v} in $CS(p) = S(p) [\bar{v} - (p + \bar{c})]$ yields

$$CS(p) = \frac{S(p)}{1 - \xi} [\sigma + \xi(p + \bar{c} - \mu)] \quad (5)$$

So the effect of a small *tightening* of a price control on aggregate consumer welfare is

$$-CS'(p) = \frac{-S'(p)}{1 - \xi} [\sigma + \xi(p + \bar{c} - \mu)] - \frac{S(p)}{1 - \xi} \xi \left(1 + \frac{d\bar{c}}{dp}\right)$$

Noting $\sigma + \xi(p - \mu) = -p/\eta$ and $\varphi = pS'(p)/S(p)$, gives

$$-CS'(p) = \frac{S(p)}{1 - \xi} \left[\frac{\varphi}{\eta} \left(1 - \frac{\eta\xi\bar{c}}{p}\right) - \xi \left(1 + \frac{d\bar{c}}{dp}\right) \right] \quad (6)$$

Welfare Effects with No Rent-Seeking: With random allocation without rent-seeking $\bar{c} = \frac{d\bar{c}}{dp} = 0$, so consumers gain from a tighter price control if and only if $\xi\eta > \varphi$.

Welfare Effects with Rent-Seeking: With rent-seeking we know from Proposition 3 that consumers must lose from any tighter control if $\xi \geq 0$, and it is clear from (5) that consumers' total surplus is always lower with rent-seeking than without if $\xi < 0$. So the conditions for consumers to gain from *any* price cut from the market price are *always* tighter with rent-seeking than without, in this class of demands.

Welfare Effects of Partial Decontrol: Because $MR(v)$ is affine in v for GPDs, the mathematics of partial decontrol are the same: in this case, p is the *average* cash price paid for units, including both those controlled and decontrolled. So from (5), when supply is inelastic, the average "effective total price" to consumers, $p + \bar{c}(p)$, is a sufficient statistic for the effect of rationing on them, that is, the effect of any change in cost to purchasers is independent of whether it is due to a partial control, or a change in rent-seeking, or both.³²

³²However, the amount of rent-seeking generally depends on the *distribution* of controlled prices, not just on the average price, p , and supply may do so too. So S , S' , \bar{c}' , and hence $CS'(p)$, generally depend on how this distribution changes.

5 Conclusion

Price controls lead to inefficient allocation and rent-seeking, in addition to reduced supply. Even absent any supply effect, inefficient allocation may cost consumers all the surplus gains they receive from a lower price and more. The results apply whether the good is allocated randomly through a lottery without rent-seeking costs, or whether greater search and other rent-seeking activities undertaken by higher-value consumers results in a more-efficient-than-random allocation. The results also apply when only some units are allocated at below-market prices, while other are sold on the free market.

In short, and especially if supply is fairly elastic, it is unlikely we can be confident that consumer surplus is enhanced by any price control.

Appendix

A. More-Efficient-than-Random Rationing with No rent-seeking in the General Case

At the market-clearing price, a \$1 price cut increases consumer surplus by \$1 for each of the $\lambda S(p)$ efficiently-allocated units, so (3) becomes

$$-CS'(p) = (1 - \lambda)\left\{-D'(p)\frac{S(p)}{D(p)}[MCS(p)] + [D'(p)\frac{S(p)}{D(p)} - S'(p)][ACS(p)]\right\} + \lambda\{S(p)\}.$$

Dividing the right hand side by $D(p) = S(p)$ (and reorganising, recalling $MCS(p) = -D(p)/D'(p)$), yields

$$\text{sign}[-CS'(p)] = \text{sign}\left[1 - (1 - \lambda)\left(\left|\frac{\text{Elasticity of Supply}}{\text{Elasticity of Demand}}\right| + 1\right)\frac{ACS(p)}{MCS(p)}\right]$$

So if demand is convex, then since (as we noted in the main text) $\frac{ACS(p)}{MCS(p)} > \frac{1}{2}$ we have $-CS'(p) < 0$ if $\left|\frac{\text{Elasticity of Supply}}{\text{Elasticity of Demand}}\right| > \frac{1+\lambda}{1-\lambda}$, while for any log-convex demand $\frac{ACS(p)}{MCS(p)} > 1$ so $-CS'(p) < 0$ if $\left|\frac{\text{Elasticity of Supply}}{\text{Elasticity of Demand}}\right| > \frac{\lambda}{1-\lambda}$.

B. Non-random Rationing with No rent-seeking in the GPD Case

Continuing the GPD example of Section 4, if fraction λ of the supply is allocated perfectly efficiently among fraction λ of the market, (6) together with the fact that a \$1 price cut increases consumer surplus by \$1 per efficiently-allocated unit at market-clearing, implies

$$-CS'(p) = S(p)\left[\frac{(1-\lambda)}{(1-\xi)}\left[\frac{\varphi}{\eta} - \xi\right] + \lambda\right] = \frac{S(p)}{1-\xi}\left[\left(\frac{\varphi}{\eta} - \xi\right) + \lambda\left(1 - \frac{\varphi}{\eta}\right)\right]$$

So consumers gain from a price reduction iff $\lambda > \frac{\varphi - \xi\eta}{\varphi - \eta}$. For example, with constant-elasticity demand this requires $\lambda > \frac{\varphi + 1}{\varphi - \eta}$.

If also fraction θ of supply is allocated as *inefficiently* as possible above the controlled price among fraction θ of the total market, then for this fraction, a \$1 price cut from the market-clearing price increases consumer surplus by \$1 per customer ($\theta S(p)$ in all), but removes $\theta(S'(p) - D'(p)) = \theta(\varphi - \eta)S(p)/p$

units from the highest-value consumers, costing $[(\mu\xi - \sigma)/\xi] - p = p/\eta\xi$ each if $\xi < 0$. (If $\xi \geq 0$, the highest-value consumer has $v = \infty$, so $-CS'(p) \rightarrow \infty$ for any $\theta > 0$.) So

$$-CS'(p) = S(p) \left[\frac{(1 - (\lambda + \theta))}{(1 - \xi)} \left[\frac{\varphi}{\eta} - \xi \right] + \lambda + \theta \left(1 + \frac{1}{\xi} \left(1 - \frac{\varphi}{\eta} \right) \right) \right] \quad \text{if } \xi < 0$$

which simplifies to

$$-CS'(p) = \frac{S(p)}{1 - \xi} \left[\left(\frac{\varphi}{\eta} - \xi \right) + \left(\lambda + \frac{\theta}{\xi} \right) \left(1 - \frac{\varphi}{\eta} \right) \right] \quad \text{if } \xi < 0.$$

When $\theta = 1$, $-CS'(p) = \frac{S(p)}{\xi(1-\xi)} \left[1 + \xi - \frac{\varphi}{\eta} \right] > 0$ for all $\xi < -1$ if $\varphi = 0$. So, for example, with linear demand consumer surplus is enhanced by a price control however inefficiently supply is allocated, if there are neither supply effects nor rent-seeking costs.

C. Dynamic Model of Incumbents and Newcomers

Assume the price falls gradually from the market level, p^M , asymptoting to $p^M - \Delta$, so the controlled price at time t is $p(t) = (p^M - \Delta) + \Delta e^{-zt}$. Consumers leave at rate δ , and are replaced by new consumers with values drawn from the distribution corresponding to demand $D(\cdot)$, using random rationing (without rent-seeking costs) among all potential consumers who wish to purchase at that time. The continuous interest rate is r . Assume GPD demand.

The surplus gain per time-0 incumbent equals the present value (to infinity) of an immediate rent reduction of Δ less the present value of the excess above $p^M - \Delta$ that is paid as prices gradually fall, that is, $\frac{\Delta}{r+\delta} - \frac{\Delta}{r+\delta+z} = \frac{z\Delta}{(r+\delta)(r+\delta+z)}$.

The present value of price cuts, as of time t , to a newcomer who buys at time t , is $\frac{\Delta}{r+\delta} - \frac{\Delta e^{-zt}}{r+\delta+z}$, so the present value of all future price cuts is $\int_0^{\infty} \delta \left[\frac{\Delta}{r+\delta} - \frac{\Delta e^{-zt}}{r+\delta+z} \right] e^{-rt} dt = \frac{\delta z \Delta}{(r+\delta)(r+z)} \left(\frac{2r+\delta+z}{r(r+\delta+z)} \right)$. Since the prices consumers pay at any time equal the average of their MRs , and since for the GPD a \$1 dollar increase in the average of consumers' MRs implies a corresponding $\xi/(1-\xi)$ average increase in their average welfare, the change in the present value of surplus for future consumers is $\frac{-\xi}{1-\xi} \left(\frac{\delta z \Delta}{(r+\delta)(r+z)} \right) \left(\frac{2r+\delta+z}{r(r+\delta+z)} \right)$.

The ratio of surplus gained by future consumers to that gained by incumbents is therefore $\frac{-\xi}{1-\xi} \frac{\delta}{r} \left(\frac{2r+\delta+z}{r+z} \right) : 1$.

For calibration, the 2007 American Housing Survey (e.g., Table 4-12) estimates that 12.4 million out of 35.0 million renters moved in the previous year, which would correspond to a continuous hazard rate of $\delta = -\ln\left(\frac{35-12.4}{35}\right) = .43$. So, for example, with demand of constant elasticity, η , $r = .02$ (real interest rate of 2%) and $z = .2$ (so half the ultimate price reduction takes place in $(-\ln(1/2)/.2) \approx 3.5$ years), then the ratio of newcomers' surplus loss to incumbents' gain $\approx 65 : -(\eta + 1)$.³³

D. Proof of Proposition 4

The price control removes some consumers with higher values than the market clearing price, p^M , and adds some lower-value consumers. As noted in the main text, convexity implies $\frac{MCS(p)}{MCS(p^M)} \geq \frac{D(p)}{D(p^M)}$ if $p > p^M$, with equality when demand is linear. So removing the high-value consumers has a more negative impact on consumer welfare than would removing the same consumers from the linear demand that is tangent to our demand at p^M (by "same" consumers, we mean those whose rank-order in the distribution of values is the same, i.e, those for whom $D(v)$ is the same). Likewise, adding the low-value consumers has a less positive impact (since $\frac{MCS(p)}{MCS(p^M)} \leq \frac{D(p)}{D(p^M)}$ if $p < p^M$).

Furthermore, rent-seeking costs that are uncorrelated with values lead to less substitution of high- v by low- v consumers than in a random allocation. So the specified substitutions would have a less beneficial impact on consumer welfare in the linear demand case, than if the consumers to be added and removed were selected randomly from those above the controlled price (since for linear demand $MCS(p)$ decreases as p falls). But, by Proposition 2, the random selection/linear demand case hurts consumers. \square

³³The calculation is purely illustrative! Issues include: using declining hazard rates with the same average tenure would reduce the ratio. We have also not accounted for any surplus that current incumbents may expect to receive in future roles as newcomers. And if it is difficult to re-enter the market to obtain a new apartment, turnover rates will be lower than in an uncontrolled market. On the other hand, these lower turnover rates are caused by tenants whose values are at least below the market price, and may be below the controlled price if they are uncertain about their future values. Furthermore, if expected turnover rates differ, consumers with longer expected residence will jump in to the market sooner, reducing the efficiency of rationing among newcomers, and further reducing their welfare.

E. Partially Decontrolled Markets

Let q_i units be rationed at price p_i , with $p_n < p_{n-1} < \dots < p_1$. Let the equilibrium uncontrolled market price be p_0 , and the equilibrium effort required to obtain a unit at price p_i be e_i . Clearly $e_n > e_{n-1} > \dots > e_1 > e_0 = 0$, and consumers sort themselves so that those with costs of effort $c \in (c_{i+1}, c_i)$ buy a unit at price p_i , where $c_i = (p_{i-1} - p_i)/(e_i - e_{i-1})$ (defining $c_{n+1} = 0$), iff their value also exceeds $p_i + e_i c$; those with costs of effort above c_1 buy an uncontrolled unit iff their value exceeds p_0 .

To see that there is generally a unique equilibrium for any given $\{p_1, \dots, p_n, q_1, \dots, q_n\}$, observe that the values of all of the c_i and e_i can be determined sequentially from c_n , that a lower c_n implies that all of the c_i and e_i are lower, and so also $p_0 (= p_1 + e_1 c_1)$ is lower, and so (since p_0 and c_1 are both lower) increases demand for uncontrolled units but must (weakly) reduce their supply; so there is generally a unique c_n for which demand equals supply and which is therefore consistent with equilibrium.

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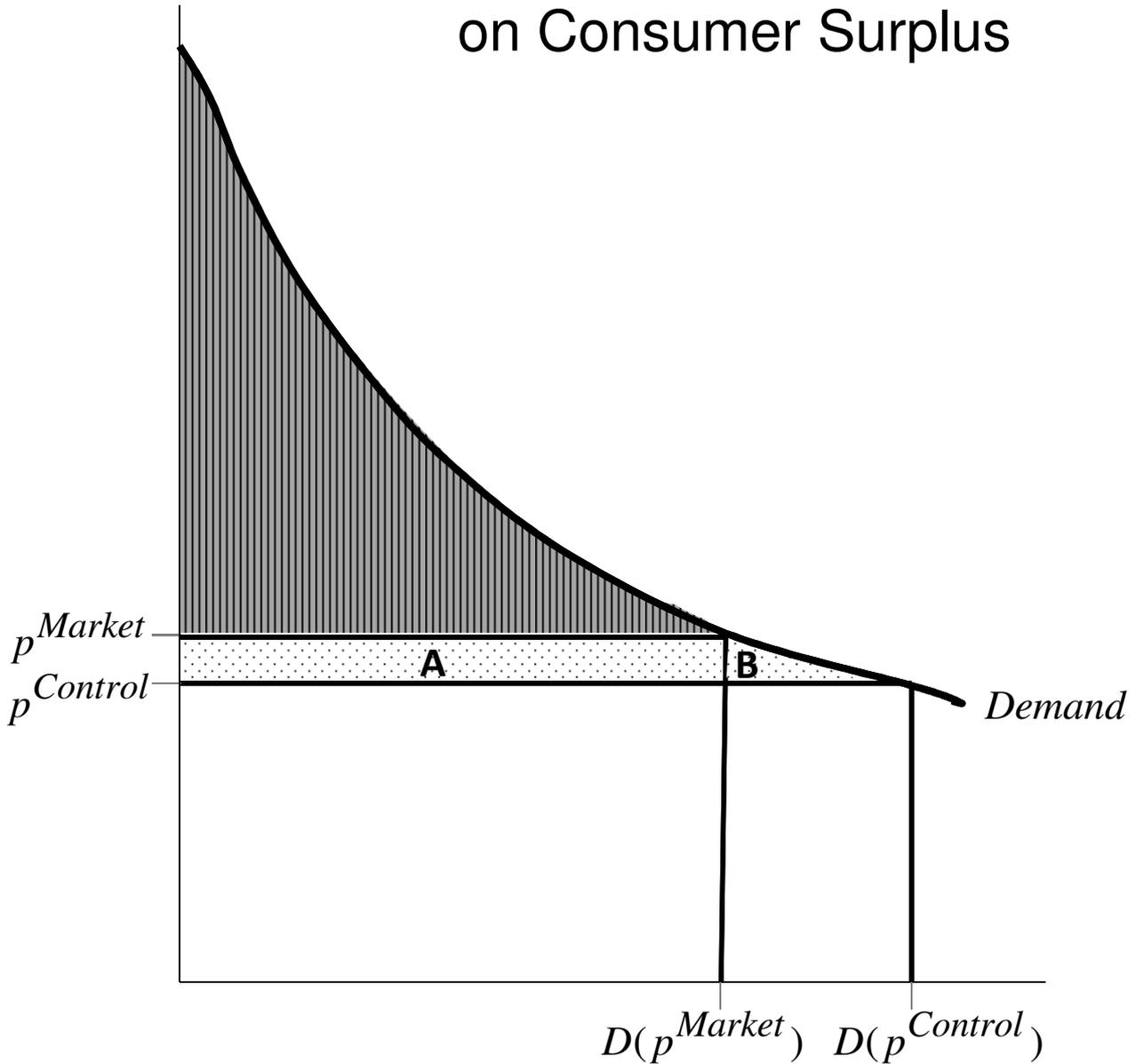
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Figure 1A

Effect of a Price Control on Consumer Surplus



 Consumer surplus at the uncontrolled price, p^{Market}

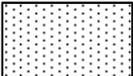
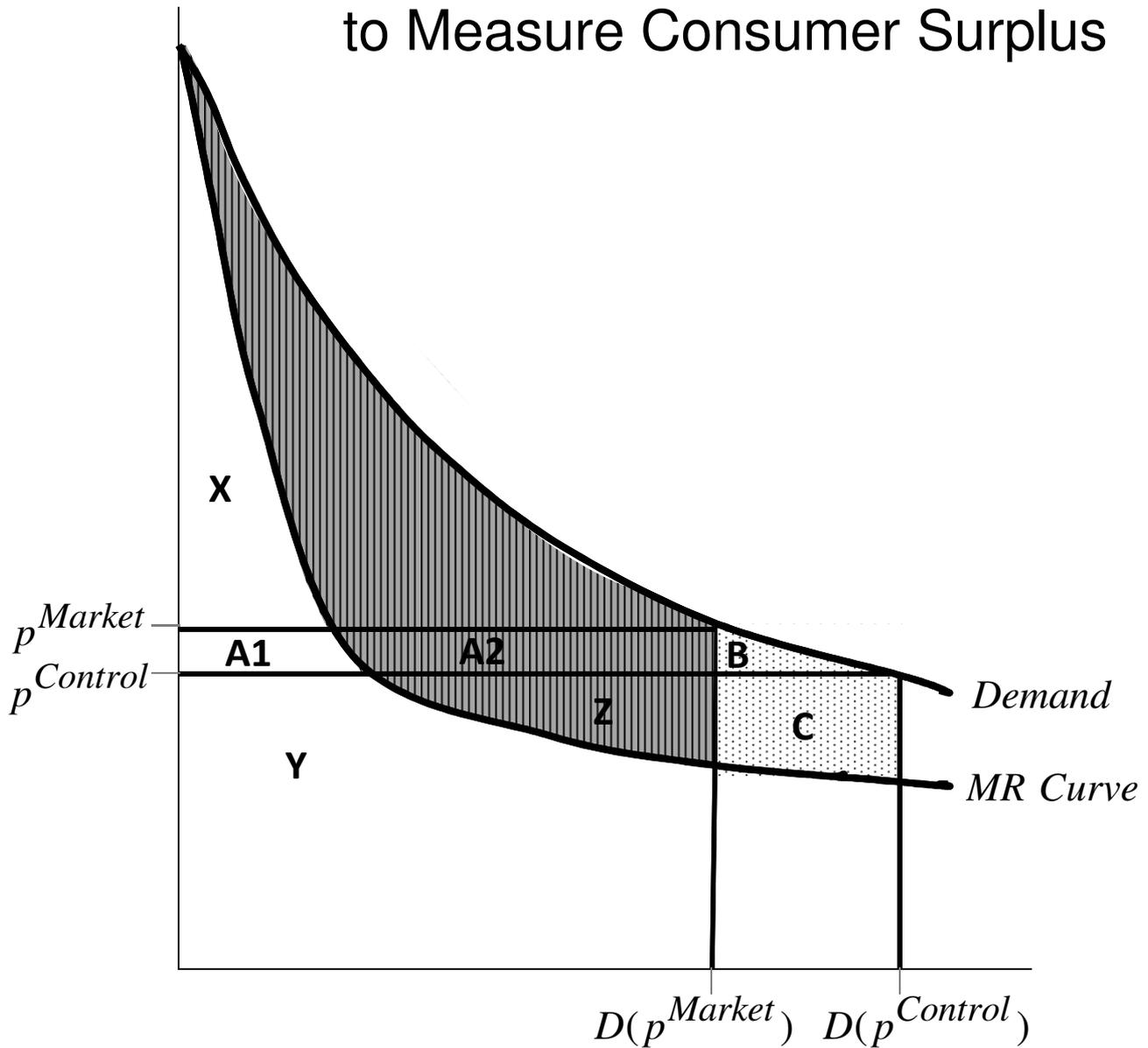
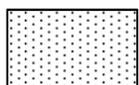
 Incremental consumer surplus that would be created **if** all consumers who wished to could purchase at the controlled price, $p^{Control}$

Figure 1B

Using Marginal Revenue Curve to Measure Consumer Surplus



Consumer surplus at the uncontrolled price, p^{Market}



Incremental consumer surplus that would be created **if** all consumers who wished to could purchase at the controlled price, $p^{Control}$

[Of which: { area B would go to new consumers
 { area C (= A1+A2 = A) goes to existing consumers}]